## Robust numerical method for singularly perturbed differential equations having both large and small delay

Robust numerical method

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#### Abstract

Purpose – The purpose of this study is to develop stable, convergent and accurate numerical method for solving singularly perturbed differential equations having both small and large delay.

Design/methodology/approach — This study introduces a fitted nonpolynomial spline method for singularly perturbed differential equations having both small and large delay. The numerical scheme is developed on uniform mesh using fitted operator in the given differential equation.

Findings — The stability of the developed numerical method is established and its uniform convergence is proved. To validate the applicability of the method, one model problem is considered for numerical experimentation for different values of the perturbation parameter and mesh points.

Originality/value – In this paper, the authors consider a new governing problem having both small delay on convection term and large delay. As far as the researchers' knowledge is considered numerical solution of singularly perturbed boundary value problem containing both small delay and large delay is first being considered.

Keywords Singularly perturbed, Small delay, Large delay, Fitted spline, e-uniformly convergent Paper type Research paper

#### 1. Introduction

A differential equation is said to be singularly perturbed delay differential equation, if it includes at least one delay term, involving unknown functions occurring with different arguments, and also, the highest derivative term is multiplied by a small parameter. Such type of delay, differential equations play a very important role in the mathematical models of science and engineering, such as the human pupil light reflex with mixed delay type [1], variational problems in control theory with small state problem [2], models of HIV infection [3] and signal transition [4].

Any system involving a feedback control almost involves time delay. The delay occurs because a finite time is required to sense the information and then react to it. Finding the solution of singularly perturbed delay differential equations, whose application mentioned above, is a challenging problem. In response to these, in recent years, there has been a growing interest in numerical methods on singularly perturbed delay differential equations. The

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# **Singularly Perturbed Differential Equati**

Michal Rosen-Zvi

#### **Singularly Perturbed Differential Equati:**

Numerical Methods for Singularly Perturbed Differential Equations Hans-Görg Roos, Martin Stynes, Lutz Tobiska, 2013-06-29 The analysis of singular perturbed differential equations began early in this century when approximate solutions were constructed from asymptotic ex pansions Preliminary attempts appear in the nineteenth century vD94 This technique has flourished since the mid 1960s Its principal ideas and methods are described in several textbooks Nevertheless asymptotic ex pansions may be impossible to construct or may fail to simplify the given problem then numerical approximations are often the only option. The systematic study of numerical methods for singular perturbation problems started somewhat later in the 1970s While the research frontier has been steadily pushed back the exposition of new developments in the analysis of numerical methods has been neglected Perhaps the only example of a textbook that concentrates on this analysis is DMS80 which collects various results for ordinary differential equations but many methods and techniques that are relevant today especially for partial differential equa tions were developed after 1980 Thus contemporary researchers must comb the literature to acquaint themselves with earlier work Our purposes in writing this introductory book are twofold First we aim to present a structured account of recent ideas in the numerical analysis of singularly perturbed differential equations Second this important area has many open problems and we hope that our book will stimulate further investigations Our choice of topics is inevitably personal and reflects our own main interests Fitted Numerical Methods For Singular Perturbation Problems: Error Estimates In The Maximum Norm For Linear Problems In One And Two Dimensions (Revised Edition) John J H Miller, Eugene O'riordan, G I Shishkin, 2012-02-29 Since the first edition of this book the literature on fitted mesh methods for singularly perturbed problems has expanded significantly Over the intervening years fitted meshes have been shown to be effective for an extensive set of singularly perturbed partial differential equations In the revised version of this book the reader will find an introduction to the basic theory associated with fitted numerical methods for singularly perturbed differential equations Fitted mesh methods focus on the appropriate distribution of the mesh points for singularly perturbed problems The global errors in the numerical approximations are measured in the pointwise maximum norm The fitted mesh algorithm is particularly simple to implement in practice but the theory of why these numerical methods work is far from simple This book can be used as an introductory text to the theory The Theory of Singular Perturbations E.M. de Jager, J.F. Furu, 1996-11-08 The underpinning fitted mesh methods subject of this textbook is the mathematical theory of singular perturbations which despite its respectable history is still in a state of vigorous development Singular perturbations of cumulative and of boundary layer type are presented Attention has been given to composite expansions of solutions of initial and boundary value problems for ordinary and partial differential equations linear as well as quasilinear also turning points are discussed The main emphasis lies on several methods of approximation for solutions of singularly perturbed differential equations and on the mathematical justification of these

methods The latter implies a priori estimates of solutions of differential equations this involves the application of Gronwall s lemma maximum principles energy integrals fixed point theorems and G ding s theorem for general elliptic equations These features make the book of value to mathematicians and researchers in the engineering sciences interested in the mathematical justification of formal approximations of solutions of practical perturbation problems. The text is selfcontained and each chapter is concluded with some exercises **Singular Perturbation Methods for Ordinary Differential Equations** Robert E., Jr. O'Malley, 2012-12-06 This book results from various lectures given in recent years Early drafts were used for several single semester courses on singular perturbation meth ods given at Rensselaer and a more complete version was used for a one year course at the Technische Universitat Wien Some portions have been used for short lecture series at Universidad Central de Venezuela West Vir ginia University the University of Southern California the University of California at Davis East China Normal University the University of Texas at Arlington Universita di Padova and the University of New Hampshire among other places As a result I ve obtained lots of valuable feedback from students and listeners for which I am grateful This writing continues a pattern Earlier lectures at Bell Laboratories at the University of Edin burgh and New York University and at the Australian National University led to my earlier works 1968 1974 and 1978 All seem to have been useful for the study of singular perturbations and I hope the same will be true of this monograph I ve personally learned much from reading and analyzing the works of others so I would especially encourage readers to treat this book as an introduction to a diverse and exciting literature The topic coverage selected is personal and reflects my current opin ions An attempt has been made to encourage a consistent method of ap proaching problems largely through correcting outer limits in regions of rapid change Formal proofs of correctness are not emphasized **Singularly Perturbed Differential** Equations Herbert Goering, Andreas Felgenhauer, Gert Lube, Hans-Görg Roos, Lutz Tobiska, 1984-01-14 No detailed description available for Singularly Perturbed Differential Equations **Robust Numerical Methods for Singularly Perturbed Differential Equations** Hans-Görg Roos, Martin Stynes, Lutz Tobiska, 2008-09-17 This new edition incorporates new developments in numerical methods for singularly perturbed differential equations focusing on linear convection diffusion equations and on nonlinear flow problems that appear in computational fluid dynamics Singular Perturbations and Hysteresis Michael P. Mortell, Robert E. O'Malley, Alexei Pokrovskii, Vladimir Sobolev, 2005-06-01 This book unifies many important recent developments in the analysis of singular perturbation and hysteresis phenomena in an accessible and comprehensive fashion In April 2002 at University College Cork in Ireland the editors conducted a workshop to provide a forum for experts to share their interests and knowledge For this book the editors have compiled research from those practitioners in areas such as reacting systems semiconductor lasers shock phenomena in economic modeling and fluid mechanics all with an emphasis on hysteresis and singular perturbations A basic introduction to hysteresis and singular perturbation theory is included with simple examples from both physics and mathematics Later chapters address applications

of hysteresis to economics various aspects of the asymptotic theory of singularly perturbed systems typical problems of the asymptotic theory of contrast structures and the geometrical approach to an investigation of models with singular perturbations and hysteresis Singularly Perturbed Differential Equations Herbert Goering, 1983 Algebraic Analysis of Singular Perturbation Theory Takahiro Kawai, Yoshitsugu Takei, 2005 The topic of this book is the study of singular perturbations of ordinary differential equations i e perturbations that represent solutions as asymptotic series rather than as analytic functions in a perturbation parameter The main method used is the so called WKB Wentzel Kramers Brillouin method originally invented for the study of quantum mechanical systems The authors describe in detail the WKB method and its applications to the study of monodromy problems for Fuchsian differential equations and to the analysis of Painleve functions This volume is suitable for graduate students and researchers interested in differential equations and special functions Introduction to the General Theory of Singular Perturbations S. A. Lomov, This book is aimed at researchers and students in physics mathematics and engineering It contains the first systematic presentation of a general approach to the integration of singularly perturbed differential equations describing nonuniform transitions such as the occurrence of a boundary layer discontinuities boundary effects and so on The method of regularization of singular perturbations presented here can be applied to the asymptotic integration of systems of ordinary and partial differential equations

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